

# ANDHRA PRADESH STATE COUNCIL OF HIGHER EDUCATION

(A Statutory body of the Government of Andhra Pradesh)

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# REVISED SYLLABUS OF B.A. /B.Sc. MATHEMATICS UNDER CBCS FRAMEWORK WITH EFFECT FROM 2020-2021

# PROGRAMME: THREE-YEAR B.A. /B.Sc. MATHEMATICS

 (With Learning Outcomes, Unit-wise Syllabus, References, Co-curricular Activities & Model Q.P.) For Fifteen Courses of 1, 2, 3 & 4 Semesters)
 (To be Implemented from 2020-21 Academic Year)

# A.P. STATE COUNCIL OF HIGHER EDUCATION

# **B.A./B.Sc. MATHEMATICS**

# **REVISED SYLLABUS FOR CORE COURSES**

# **CBCS/ SEMESTER SYSTEM**

# (w.e.f. 2020-21 Admitted Batch)

# CORE COURSES STRUCTURE

# (Sem-I to Sem-IV)

Course	Subject	Hrs.	Credits	IA	ES	Total
Course -I	Differential Equations & Differential Equations Problem Solving Sessions	6	5	25	75	100
Course -II	Three dimensional analytical Solid geometry & Three dimensional analytical Solid Geometry Problem Solving Sessions	6	5	25	75	100
Course -III	Abstract Algebra & Abstract Algebra Problem Solving Sessions	6	5	25	75	100
Course -IV	Real Analysis & Real Analysis Problem Solving Sessions	6	5	25	75	100
Course -V	Linear Algebra & Linear Algebra Problem Solving Sessions	6	5	25	75	100

# COURSE-I CBCS/ SEMESTER SYSTEM B.A./B.Sc. MATHEMATICS (w.e.f. 2020-21 Admitted Batch) DIFFERENTIAL EQUATIONS SYLLABUS (75 Hours)

## **Course Outcomes:**

After successful completion of this course, the student will be able to;

- 1. Solve linear differential equations
- 2. Convertnonexact homogeneous equations to exact differential equations by using integrating factors.
- 3.Know the methods of finding solutions of differential equations of the firstorder but not of the first degree.
- 4 Solvehigher-order linear differential equations, both homogeneous and non homogeneous, with constant coefficients.
- 5 Understand the concept and apply appropriate methods for solving differential equations.

# **Course Syllabus:**

# UNIT – I (12 Hours)

# Differential Equations of first order and first degree:

Linear Differential Equations; Differential equations reducible to linear form; Exact differential equations; Integrating factors; Change of variables.

# UNIT – II (12 Hours)

# **Orthogonal Trajectories**

# Differential Equations of first order but not of the first degree:

Equations solvable for p; Equations solvable for y; Equations solvable for x; Equations that do not contain x (or y); Equations homogeneous in x and y; Equations of the first degree in x and y – Clairaut's Equation.

# UNIT - III (12 Hours)

# Higher order linear differential equations-I:

Solution of homogeneous linear differential equations of order n with constant coefficients; Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators. General Solution of f(D)y=0.

General Solution of f(D)y=Q when Q is a function of x,  $\frac{1}{f(D)}$  is expressed as partial fractions.

P.I. of f(D)y = Q when  $Q = be^{ax}$ 

P.I. of f(D)y = Q when Q is beinax or b cos ax.

#### UNIT - IV (12 Hours)

# Higher order linear differential equations-II:

Solution of the non-homogeneous linear differential equations with constant coefficients.

P.I. of f(D)y = Q when  $Q = bx^k$ 

P.I. of f(D)y = Q when  $Q = e^{ax} V$ , where V is a function of x.

of f(D)y = Q when Q = xV, where V is a function of x.

of f(D)y = Q when  $Q = x^m V$ , where V is a function of x.

# UNIT -V (12 Hours)

# Higher order linear differential equations-III :

Method of variation of parameters; Linear differential Equations with non-constant coefficients; The Cauchy-Euler Equation, Legendre's linear equations.

# **Co-Curricular Activities(15 Hours)**

Seminar/ Quiz/ Assignments/ Applications of Differential Equations to Real life Problem /Problem Solving.

# **Text Book :**

Differential Equations and Their Applications by Zafar Ahsan, published by Prentice-Hall of India Pvt. Ltd, New Delhi-Second edition.

# **Reference Books :**

- A text book of Mathematics for B.A/B.Sc, Vol 1, by N. Krishna Murthy & others, published by S.Chand & Company, New Delhi.
- 2. Ordinary and Partial Differential Equations by Dr. M.D,Raisinghania, published by S. Chand & Company, New Delhi.
- 3.Differential Equations with applications and programs S. Balachandra Rao & HR Anuradha-Universities Press.
- 4. Differential Equations -Srinivas Vangala & Madhu Rajesh, published by Spectrum University Press.

# COURSE-II CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY Syllabus (75 Hours)

#### **Course Outcomes:**

After successful completion of this course, the student will be able to;

- 1. get the knowledge of planes.
- 2. basic idea of lines, sphere and cones.
- 3. understand the properties of planes, lines, spheres and cones.
- 4. express the problems geometrically and then to get the solution.

## **Course Syllabus:**

#### UNIT – I (12 Hours)

#### The Plane :

Equation of plane in terms of its intercepts on the axis, Equations of the plane through the given points, Length of the perpendicular from a given point to a given plane, Bisectors of angles between two planes, Combined equation of two planes.

#### UNIT – II (12 hrs)

# The Line :

Equation of a line; Angle between a line and a plane; The condition that a given line may lie in a given plane; The condition that two given lines are coplanar; Number of arbitrary constants in the equations of straight line; Sets of conditions which determine a line; The shortest distance between two lines; The length and equations of the line of shortest distance between two straight lines; Length of the perpendicular from a given point to a given line.

# UNIT - III (12 hrs)

# The Sphere :

Definition and equation of the sphere; Equation of the sphere through four given points; Plane sections of a sphere; Intersection of two spheres; Equation of a circle; Sphere through a given circle;

Intersection of a sphere and a line; Power of a point; Tangent plane; Plane of contact; Polar plane; Pole of a Plane; Conjugate points; Conjugate planes;

# UNIT - IV (12 hrs)

# The Sphere and Cones :

Angle of intersection of two spheres; Conditions for two spheres to be orthogonal; Radical plane; Coaxial system of spheres; Simplified from of the equation of two spheres.

Definitions of a cone; vertex; guiding curve; generators; Equation of the cone with a given vertex and guiding curve; equations of cones with vertex at origin are homogenous; Condition that the general equation of the second degree should represent a cone;

#### UNIT - V (12 hrs)

#### **Cones** :

Enveloping cone of a sphere; right circular cone: equation of the right circular cone with a given vertex, axis and semi vertical angle: Condition that a cone may have three mutually perpendicular generators; intersection of a line and a quadric cone; Tangent lines and tangent plane at a point; Condition that a plane may touch a cone; Reciprocal cones; Intersection of two cones with a common vertex.

# **Co-Curricular Activities(15 Hours)**

Seminar/ Quiz/ Assignments/Three dimensional analytical Solid geometry and its applications/ Problem Solving.

#### **Text Book :**

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand & Company Ltd. 7th Edition.

# **Reference Books :**

- A text book of Mathematics for BA/B.Sc Vol 1, by V Krishna Murthy & Others, published by S. Chand & Company, New Delhi.
- 2. A text Book of Analytical Geometry of Three Dimensions, by P.K. Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.
- Co-ordinate Geometry of two and three dimensions by P. Balasubrahmanyam, K.Y. Subrahmanyam, G.R. Venkataraman published by Tata-MC Gran-Hill Publishers Company Ltd., New Delhi.
- 4. Solid Geometry by B.Rama Bhupal Reddy, published by Spectrum University Press.

# COURSE-III CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS ABSTRACT ALGEBRA SYLLABUS (75 Hours)

# **Course Outcomes:**

After successful completion of this course, the student will be able to;

- 1. acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
- 2. get the significance of the notation of a normal subgroups.
- 3. get the behavior of permutations and operations on them.
- 4. study the homomorphisms and isomorphisms with applications.
- 5. understand the ring theory concepts with the help of knowledge in group theory and to prove the theorems.
- 6. understand the applications of ring theory in various fields.

# **Course Syllabus:**

# UNIT - I (12 Hours)

#### **GROUPS**:

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

# UNIT - II (12 Hours)

#### **SUBGROUPS**:

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definitionexamples-criterion for a complex to be a subgroups. Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups.

# **Co-sets and Lagrange's Theorem :**

Cosets Definition – properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

## UNIT -III (12 Hours)

#### **NORMAL SUBGROUPS :**

**HOMOMORPHISM:** 

Definition of normal subgroup – proper and improper normal subgroup–Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group.

# UNIT - IV (12 Hours)

Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

## UNIT - V (12 Hours)

## **RINGS**:

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings, Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field, Sub Rings.

#### **Co-Curricular Activities(15 Hours)**

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

## **Text Book :**

A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published by S.Chand & Company, New Delhi.

# **Reference Books :**

- 1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
- 2. Modern Algebra by M.L. Khanna.
- 3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

# COURSE-IV CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS REAL ANALYSIS SYLLABUS (75 Hours)

# **Course Outcomes:**

After successful completion of this course, the student will be able to

- 1. get clear idea about the real numbers and real valued functions.
- obtain the skills of analyzing the concepts and applying appropriate methods for testing convergence of a sequence/ series.
- 3. test the continuity and differentiability and Riemann integration of a function.
- 4. know the geometrical interpretation of mean value theorems.

# **Course Syllabus:**

# UNIT - I (12Hours)

# **REAL NUMBERS :**

The algebraic and order properties of R, Absolute value and Real line, Completeness property of R, Applications of supremum property; intervals. Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence.

# (No question is to be set from this portion).

# **INFINITIE SERIES :**

**Series :**Introduction to series, convergence of series. Cauchy's general principle of convergence for series tests for convergence of series, Series of Non-Negative Terms.

- 1. P-test
- 2. Cauchy's n<sup>th</sup> root test or Root Test.
- 3. D'-Alemberts' Test or Ratio Test.
- 4. Alternating Series Leibnitz Test.

Absolute convergence and conditional convergence.

# UNIT – II (12 Hours)

# **CONTINUITY :**

Limits : Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limit concept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).Continuous functions : Continuous functions, Combinations of continuous functions, Continuous

Functions on intervals, uniform continuity.

# UNIT - III (12 Hours)

#### **DIFFERENTIATION AND MEAN VALUE THEORMS :**

The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean value Theorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

#### UNIT - IV(12 Hours)

# **RIEMANN INTEGRATION :I**

Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Another definition of Riemann integral, Some classes of Bounded integrable functions.

# UNIT -V(12 Hours)

# **RIEMANN INTEGRATION :II**

Properties of integrals functions, Fundamental theorem of integral calculus, integral as the limit of a sum, Mean value Theorems.

## **Co-Curricular Activities(15 Hours)**

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

# **Text Book:**

Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, published by John Wiley.

#### **Reference Books:**

- 1.A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand & Company Pvt. Ltd., New Delhi.
- 2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania, published by S. Chand & Company Pvt. Ltd., New Delhi.

# COURSE-V CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS LINEAR ALGEBRA SYLLABUS (75 Hours)

#### **Course Outcomes:**

After successful completion of this course, the student will be able to;

- 1. understand the concepts of vector spaces, subspaces, basis, dimension and their properties
- 2. understand the concepts of linear transformations and their properties
- 3. understand the elementary properties of matrices and rank of matrix
- 4. apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods

## **Course Syllabus:**

# UNIT – I (12 Hours)

#### **Vector Spaces-I:**

Vector Spaces, General properties of vector spaces, n-dimensional Vectors, addition and scalar multiplication of Vectors, internal and external composition, Null space, Vector subspaces, Algebra of subspaces, Linear Sum of two subspaces, linear combination of Vectors, Linear span Linear independence and Linear dependence of Vectors.

## UNIT -II (12 Hours)

#### **Vector Spaces-II:**

Basis of Vector space, Finite dimensional Vector spaces, basis extension, co-ordinates, Dimension of a Vector space, Dimension of a subspace, Quotient space and Dimension of Quotient space.

#### UNIT -III (12 Hours)

# **Linear Transformations:**

Linear transformations, linear operators, Properties of L.T, sum and product of LTs, Algebra of Linear Operators, Range and null space of linear transformation, Rank and Nullity of linear transformations – Rank – Nullity Theorem.

# UNIT -IV (12 Hours)

# Matrices – I :

Matrices, Elementary Properties of Matrices, Rank of Matrix, Normal form, Echelon form, Inverse of a matrix by using elementary operations.

# UNIT -V (12 Hours)

# Matrices – II:

Linear Equations: System of Homogeneous and non homogeneous Linear Equations.

Characteristic equations, Characteristic Values & Vectors of a square matrix, Cayley – Hamilton Theorem and problems.

# **Co-Curricular Activities(15 Hours)**

Seminar/ Quiz/ Assignments/ Linear algebra and its applications / Problem Solving.

# **Text Book:**

Linear Algebra by J.N. Sharma and A.R. Vasista, published by Krishna Prakashan Mandir, Meerut- 250002.

#### **Reference Books :**

- 1. Matrices by Shanti Narayana, published by S.Chand Publications.
- 2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson Education (low priced edition), New Delhi.
- Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt. Ltd. 4<sup>th</sup> Edition, 2007.

# Recommended Question Paper Patterns and Models BLUE PRINT FOR QUESTION PAPER PATTERN COURSE-I, DIFFERENTIAL EQUATIONS

Unit	TOPIC	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Differential Equations of 1 <sup>st</sup> order and 1 <sup>st</sup> degree	2	2	30
п	Orthogonal Trajectories, Differential Equations of 1 <sup>st</sup> order but not of 1 <sup>st</sup> degree	2	2	30
ш	Higher Order Linear Differential Equations (with constant coefficients) – I	2	2	30
IV	Higher Order Linear Differential Equations (with constant coefficients) – II	2	2	30
V	Higher Order Linear Differential Equations (with non constant coefficients)	2	2	30
	TOTAL	10	10	150

S.A.Q.	<b>S.A.Q.</b> = Short answer questions		(5 marks)	
E.Q.	= Essay questions		(10 marks)	
Short answ	ver questions	: 5 X 5	M = 25 M	
Essay que	stions	: 5 X 10	M = 50 M	
	Total Marks		= 75 M	

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# CBCS/ SEMESTER SYSTEM (W.e.f 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS COURSE-I, DIFFERENTIAL EQUATIONS

#### MATHEMATICS MODEL PAPER

Time: 3Hrs

Max.Marks:75M

# **SECTION - A**

#### Answer any **FIVE** questions. Each question carries **FIVE** marks5 X 5 M=25 M

1. Solve  $(1 + e^{x/y}) dx + e^{x/y}$   $(1 - \frac{x}{y}) dy = 0.$ 

2. Solve 
$$(y - e^{\sin^{-1}x}) \stackrel{dx}{\xrightarrow{dy}} \sqrt{1 - x^2} = 0$$

- 3. Solve  $y + px = p^2x^4$ .
- 4. Solve (px y)(py + x) = 2p
- 5. Solve  $(D^2 3D + 2) = \cosh x$
- 6. Solve  $(D^2 + 9) = e^x \cos 2x$
- 7. Solve $(D^2 4D + 3)y = \sin 3x \cos 2x$ .
- 8. Solve  $d^{2y} 6 d^{2y} \pm 13y = 8e^{3x} \sin 2x$ .
- 9. Solve  $x^2y'' 2x(1+x)y' + 2(1+x)y = x^3$
- 10. Solve  $[(5+2x)^2D^2 6(5+2x)D + 8]y = 0$

# **SECTION - B**

# Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. $5 \times 10 \text{ M} = 50 \text{ M}$

11. Solve 
$$x \frac{dy}{dx} + y = y^2 \log x$$
.  
(Or)  
12. Solve  $(y + \frac{1}{3}y^3 + \frac{1}{4}x^2) dx + \frac{1}{4}(x + xy^2) dy = 0$ .

13. Solvep<sup>2</sup> + 2pycotx =  $y^2$ .

14. Find the orthogonal trajectories of the family of curves

$$x^{2/3} + y^{2/3} = a^{2/3}$$
 where 'a' is the parameter.

15. Solve $(D^3 + D^2 - D - 1)y = \cos 2x$ .

(Or)

16.Solve $(D^2 - 3D + 2)y = \sin e^{-x}$ .

17.Solve 
$$(D^2 - 2D + 4)y = 8(x^2 + e^{2x} + \sin 2x)$$
  
(Or)  
18.  $\frac{d^{2y}}{dx^2} + 3 \frac{dy}{dx} + 2y = xe^x \sin x$ 

19.

Solve  $(D^2 - 2D)y = e^x \sin x$  by the method of variation of parameters.

(Or)

20. Solve 
$$3x^2 \frac{d^2y}{dx^2} + \frac{x}{dx^2} \frac{dy}{dx} + \frac{y}{dx} = x$$

# BLUE PRINT FOR QUESTION PAPER PATTERN COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Unit	TOPIC	S.A.Q(including	E.Q(including	Total Marks
Unit	TOPIC	choice)	choice)	
Ι	The Plane	2	2	30
II	The Right Line	2	2	30
III	The Sphere	2	2	30
IV	The Sphere & The Cone	2	2	30
V	The Cone	2	2	30
	TOTAL	10	10	150

S.A.Q.	= Short answer questions		(5 marks)	
E.Q.	= Essay questions		(10	) marks)
Short answ	ver questions	: 5 X 5	Μ	= 25 M
Essay que	stions	: 5 X 10	) M	= 50 M
	Total Marks			= 75 M
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#### **CBCS/ SEMESTER SYSTEM**

# (w.e.f. 2020-21 Admitted Batch)

## **B.A./B.Sc. MATHEMATICS**

# COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY Time: 3Hrs Max.Marks:75 M

# **SECTION - A**

#### Answer any <u>FIVE</u> questions. Each question carries <u>FIVE</u> marks 5 X 5 M=25 M

- 1. Find the equation of the plane through the point (-1,3,2) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z=8.
- 2. Find the bisecting plane of the acute angle between the planes 3x-2y-6z+2=0, -2x+y-2z-2=0.
- 3. Find the image of the point (2,-1,3) in the plane 3x-2y+z=9.
- 4. Show that the lines 2x + -4 = 0 = y + 2z and x + 3z 4 = 0, 2x + 5z - 8 = 0 are coplanar.
- 5. A variable plane passes through a fixed point (a, b, c). It meets the axes in A,B,C. Show that the centre of the sphere OABC lies on ax<sup>-1</sup>+by<sup>-1</sup>+cz<sup>-1</sup>=2.
- 6. Show that the plane 2x-2y+z+12=0 touches the sphere x<sup>2</sup>+y<sup>2</sup>+z<sup>2</sup>-2x-4y+2z-3=0 and find the point of contact.
- 7. If  $r_1, r_2$  are the radii of the two orthogonal spheres, then show that the radius of the circle of their intersection is  $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$
- 8. Find the equation to the cone which passes through the three coordinate axes and the lines  $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$  and  $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
- 9. Find the equation of the enveloping cone of the sphere  $x^2 + y^2 + z^2 + 2x 2y = 2$ with its vertex at (1, 1, 1).
- 10. Show that reciprocal cone of  $ax^2 + by^2 + cz^2 = 0$  is the cone  $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$

#### **SECTION - B**

#### Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

11. A plane meets the coordinate axes in A, B, C. If the centroid of  $\triangle ABC$  is (a,b,c), show that the equation of the plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ .

#### (OR)

12. A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of the tetrahedron OABC is  $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$ .

- 13. Find the shortest distance between the lines
  - $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}; \ \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$
- (OR) 14. Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ ;  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  are coplanar. Also find their point of intersection and the plane containing the lines.
- 15. Show that the two circles x<sup>2</sup>+y<sup>2</sup>+z<sup>2</sup>-y+2z=0, x-y+z=2;
  x<sup>2</sup>+y<sup>2</sup>+z<sup>2</sup>+x-3y+z-5=0, 2x-y+4z-1=0 lie on the same sphere and find its equation.
  - (OR)
- 16. Find the equation of the sphere which touches the plane 3x+2y-z+2=0 at (1,-2,1) and cuts orthogonally the sphere  $x^2+y^2+z^2-4x+6y+4=0$ .
- 17. Find the limiting points of the coaxial system of spheres  $x^2+y^2+z^2-8x+2y-2z+32=0$ ,  $x^2+y^2+z^2-7x+z+23=0$ .
  - (OR)
- 18. Find the equation to the cone with vertex is the origin and whose base curve is  $x^2+y^2+z^2+2ux+d=0$ .
- 19. Prove that the equation  $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$  represents a cone that touches the coordinate planes and find its reciprocal cone.

# (OR)

20. Find the equation of the sphere  $x^2+y^2+z^2-2x+4y-1=0$  having its generators parallel to the line x=y=z.

# BLUE PRINT FOR QUESTION PAPER PATTERN COURSE-III, ABSTRACT ALGEBRA

Unit	ΤΟΡΙϹ	S.A.Q(including choice)	E.Q(including choice)	Total Marks
Ι	Groups	2	2	30
II	Subgroups, Cosets & Lagrange's theorem	2	2	30
III	Normal Subgroups	2	2	30
IV	Homomorphisms	2	2	30
V	Rings	2	2	30
	Total	10	10	150

S.A.Q.	= Short answer question	ons (5 marks)
E.Q.	= Essay questions	(10 marks)
Short answ	wer questions	: 5 X 5 M = 25 M
Essay questions		: 5 X 10 M = 50 M
	Total Marks	= 75 M

# CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS

# **COURSE-III, ABSTRACT ALGEBRA**

Time: 3Hrs

Max.Marks:75M

# **SECTION - A**

# Answer any <u>FIVE</u> questions. Each question carries <u>FIVE</u> marks 5 X 5 M=25 M

1. Show that the set  $G = \{x/x = 2^a 3^b \text{ and } a, b \in Z\}$  is a group under multiplication

- 2. Define order of an element. In a group G, prove that if  $a \in G$  then  $O(a) = O(a)^{-1}$ .
- 3. If H and K are two subgroups of a group G, then prove that HK is a subgroup ⇔ HK=KH
- 4. Prove that any two Right (left) cosets of a subgroup are either identical or disjoint.
- 5. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal

subgroup.

- 6. Define Normal Subgriop and Prove that every subgroup of an abelian group is normal .
- 7. If  $G, G^1$  are two groups and  $f: G \to G^1$  is homomorphism then show that
  - (a)  $(e) = e^1 \quad \forall e \in G \ e^1 \in G^1$
  - (b)  $(a^{-1}) = [(a)]^{-1} \forall a = G$
- 8. Prove that, every homomorphism image of an abelian group is an abelian.

9. Prove that the characteristic of an integral domain is either prime or zero.

10.Prove that the intersection of two subrings of a ring is also a subring

# **SECTION - B**

# Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

11. Show that the set of n<sup>th</sup> roots of unity forms an abelian group under multiplication.

(Or) 12. In a group G, for  $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ , O(a)=5, b  $\neq$  e and  $\boldsymbol{a}\boldsymbol{b}\boldsymbol{a}^{-1} = \boldsymbol{b}^2$ . Find O(b).

The Union of two subgroups is also a subgroup ⇔ one is contained in the other.

14. State and prove Langrage's theorem.

- 15. Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.
  - (Or)
- 16. If *M*, *N* are two Normal subgroups of G such that *M* ∩ *N* = {*e*} then show that every element of M commutes with every element of N.
- 17. State and prove fundamental theorem of homomorphisms of groups.
- (Or) 18. If G and  $G^1$  are two groups and  $f: G \to G^1$  is onto Homomorphism then prove that f is Isomorphism  $\Leftrightarrow \ker f = \{e\}.$
- 19. Prove that every finite integral domain is a field.

#### (Or)

20. Prove that the characteristic of a field is either zero or prime.

# BLUE PRINT FOR QUESTION PAPER PATTERN COURSE-IV, REAL ANALYSIS

Unit	ΤΟΡΙΟ	S.A.Q(including	E.Q(including	Total Marks
		choice)	choice)	
Ι	Real Number System and Real Sequence	2	2	30
II	Infinite Series	2	2	30
III	Limits and Continuity	2	2	30
IV	Differentiation and Mean Value Theorem	2	2	30
V	Riemann Integration	2	2	30
	TOTAL	10	10	150

S.A.Q.	= Short answer ques	tions (5 marks)
E.Q.	= Essay questions	(10 marks)
Short ans	wer questions	: 5 X 5 M = 25 M
Essay questions		: 5 X 10 M = 50 M
	Total Marks	= 75 M

# CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS COURSE-IV, REAL ANALYSIS

Time: 3Hrs

Max.Marks:75M

# **SECTION - A**

## Answer any **FIVE** questions. Each question carries **FIVE** marks 5 X 5 M=25 M

1. Test the convergence of the series  $\sum_{n=1}^{\infty} (\sqrt[3]{n^3 + 1} - n)$ . 2. Test for Convergence  $\sum_{n=1}^{\infty} \frac{1}{2n+3n}$ 

3. Examine for continuity of the function f defined by f(x) = |x| + |x - 1| at x=0 and 1.

4. Show that the function f defined by  $f(x) = x^3$  is uniformly continous on  $\begin{bmatrix} -2 & 2 \end{bmatrix}$ 

# 5. Show that $f(x) = x \sin \frac{1}{x}$ , $x \neq 0$ ; f(x) = 0, x = 0 is continuous but not derivable at x=0.

6. Verify Rolle's theorem for the function  $f(x) = x^3 - 6x^2 + 11x - 6$  on [1, 3].

- 7. If  $f(x) = x^2 \forall x \in [0, 1]$  and  $p = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$  then find L(p, f) and U(p, f).
- 8. If  $f \in R[a, b]$  and *m*, M are infimum and suprimum of *f* on [a, b] then show that  $m(b-a) \leq \int_{a}^{b} f(x)dx \leq M(b-a)$

9.prove that if **f**: **[a, b]**  $\rightarrow$  **R** is continuous on [a, b] then f is R- integrable on [a, b]. 10. By considering the integral  $\int_{0}^{1} \frac{1}{1+x} dx$  show that  $bg_{2} = \int_{0}^{1} \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$ .

#### SECTION –B

#### Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

11.State and Prove Cauchy's nth root test.

(OR)

12. Test the convergence of  $\sum \frac{x^n}{x^n + a^n}$  (x > 0, a > 0)

13.Let f:  $R \rightarrow R$  be such that

$$f(x) = \frac{\sin(a+1)x + \sin x}{x} \text{ for } x < 0$$
$$= c \qquad \text{for } x = 0$$
$$= \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} \text{ for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

## (OR)

- 14. Define uniform continuity, If a function f is continuous on [a b] then f is uniformly continuous on [ a b]
- 15. Using Lagrange's theorem, show that  $x > log(1 + x) > \frac{x}{(1+x)} \forall x > 0$ . (OR)

16. State and prove Cauchy's mean value theorem.

17. State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR) 18.Prove that f(x) = sinx is integrable on  $[0, \pi/2]$  and  $\int_0^{\pi/2} sin x dx = 1$ 19. State and prove Fundamental theorm of integral calculus.

(OR)

20. Prove that 
$$\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$$
.

# BLUE PRINT FOR QUESTION PAPER PATTERN COURSE-V, LINEAR ALGEBRA

Unit	TOPIC	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
Ι	Vector spaces - I	2	2	30
II	Vector spaces - II	2	2	30
III	Linear Transformation	2	2	30
IV	Matrices - I	2	2	30
V	Matrices - II	2	2	30
Total		10	10	150

S.A.Q.	= Short answer questions		(5 marks)	
E.Q.	= Essay questions		(10 marks)	
Short ans	wer questions	: 5 X 5	M = 25 M	
Essay que	estions	: 5 X 10	0 M = 50 M	
	Total Marks		= 75 M	
			•••••	

# CBCS/ SEMESTER SYSTEM (w.e.f. 2020-21 Admitted Batch) B.A./B.Sc. MATHEMATICS COURSE-V, LINEAR ALGEBRA

Time: 3Hrs

Max.Marks:75M

# **SECTION - A**

#### Answer any **FIVE** questions. Each question carries **FIVE** marks 5 X 5 M=25 M

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of

elements of F, such that px+qy+rz=0 is a vector subspace of  $V_3(R)$ .

- 2. Define linearly independent & linearly dependent vectors in a vector space. If
  - $\alpha$ ,  $\beta$ ,  $\gamma$  are linearly independent vectors of V(R) then show that  $\alpha + \beta$ ,  $\beta + \gamma$ ,  $\gamma + \alpha$  are

also linearly independent.

- 3. Prove that every set of (n + 1) or more vectors in an n dimensional vector space is linearly dependent.
- 4. Show that any two bases of vector space (F) have the same number of elements.
- The mapping T : V<sub>3</sub>(R)→V<sub>3</sub>(R) is defined by T(x,y,z) = (x-y,x-z). Show that T is a linear transformation.
- 6. Let  $\mathbf{T}: \mathbb{R}^3 \to \mathbb{R}^2$  and  $\mathbb{H}: \mathbb{R}^3 \to \mathbb{R}^2$  be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.

7. Find the rank of the matrix A=  $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$  by reducing to normal form.

8. Reduce the matrix A= $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$  into echelon form and find its rank.

- 9. Solve the system of equations x+y-3z+2w=0, 2x-y+2z-3w=0, 3x-2y+z-4w -4x+y-3z+w=0.
- 10. If the matrix A is non-singular, show that the eigen values of  $A^{-1}$  are the reciprocals of the eigen values of A.

#### **SECTION - B**

#### Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

11.Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the necessary and sufficient condition for W to be a subspace of V is  $a, b \in F$  and  $\alpha, \beta \in V => a\alpha + b\beta \in W$ .

## (OR)

- 12. Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of  $V_3(C)$  form linearly dependent set, but any three of them are linearly independent.
- 13.Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F) then prove that W is finite dimensional and dim W≤ n.

#### (OR)

- 14.If W be a subspace of a finite dimensional vector space V(F) then Prove that  $\dim \frac{V}{W} = \dim V - \dim W.$
- 15. Find T (x, y, z) where T: R<sup>3</sup> → R is defined by T (1, 1, 1) =3, T (0, 1, -2) =1, T (0, 0, 1) = -2

(OR)

16. State and prove Rank Nullity theorem.

17. Obtain two non singular matrices P and Q Such that  $PAQ = \begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$  where

$$A = \begin{bmatrix} 3 & 2 & -1 & 3 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$
(OR)  
18. Find the inverse of  $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$  using elementary row operations

19. Solve the system  $\lambda x+y+z=0$ ,  $x+\lambda y+z=0$ ,  $x+y+\lambda z=0$ , if the system has nonzero solution.

(OR)

20. State and prove Cayley-Hamilton theorem.

# SUBJECT EXPERTS

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